Convective Heat Transfer with Buoyancy Effects from Thermal Sources on a Flat Plate

S. S. Tewari* and Y. Jaluria†
Rutgers—The State University of New Jersey, New Brunswick, New Jersey 08903

An experimental study is carried out on the thermal interaction between two finite-size heat sources, located on a flat plate that is well insulated on the back. Both the horizontal and the vertical orientations of the surface are studied by measuring the flow velocities, the temperature field, and the local heat flux. The investigation is directed at the pure natural convection circumstance (no forced flow velocity) and the buoyancy-dominated mixed-convection circumstance (presence of a relatively small forced flow velocity). Large temperature gradients occur in the vicinity of the heat sources, resulting in a substantial diffusion of heat along the plate length. However, the effect of conduction is found to be highly localized. The orientation of the surface has a very strong effect on the interaction of the wakes from the heat sources for the circumstances considered. An upstream source is found to have a very strong influence on the temperature of a downstream source in the vertical surface orientation but has a much weaker influence in the horizontal orientation. In the latter circumstance the presence of a small forced flow velocity may actually increase the temperature of a downstream source by tilting the wake from the upstream source toward the downstream source.

Nomenclature

D	= dimensionless separation distance between two heat sources, defined in Eq. (1)
d	= separation between two heat sources, shown in Fig. 1, m
Gr	= Grashof number, defined in Eq. (4)
g	= gravitational acceleration, m/s^2
h	= heat transfer coefficient, defined in Eq. (13), W/m^2-K
k	= thermal conductivity of air, W/m-K
L	= width of the heated strip, shown in Fig. 1, m
$Nu_{w/o}$	= Nusselt number for an isolated heat source, defined in Eq. (12)
Nu_w	= Nusselt number for a heat source under the influence of another heat source, defined in Eq. (12)
q_{conv}	= heat flux convected from the surface to the flow, W/m^2
q_{in}	= total heat-flux input to the heated strips due to the electrical power dissipation, W/m^2
Re	= Reynolds number, defined in Eq. (6)
T	= local temperature in the flowfield
T_s	= surface temperature, K
T_∞	= ambient temperature, K
U_∞	= forced flow velocity, m/s
V_x	= nondimensionalized v_x, defined in Eqs. (8) and (9)
$V_{x,max}$	= maximum value of V_x
V_y	= nondimensionalized v_y, defined in Eqs. (14) and (16)
$V_{y,max}$	= maximum value of V_y
v_x, v_y	= velocity components in the x and y directions, respectively, m/s
X	= dimensionless distance from the leading edge of the plate, defined in Eq. (1)
X_0	= dimensionless distance from the leading edge of the upstream strip, defined in Eq. (1)
x	= distance from the leading edge of the surface, along the surface, shown in Fig. 1, m
x_0	= distance from the leading edge of the upstream strip, along the surface, m
Y	= dimensionless y distance from the surface, defined in Eqs. (1) and (7)
y	= distance from the plate surface, shown in Fig. 1, m
β	= coefficient of thermal expansion of air, K^{-1}
ν	= kinematic viscosity of air, m^2/s
θ	= dimensionless temperature, defined in Eq. (2)
θ_s	= dimensionless surface temperature, defined in Eqs. (3) and (5)

Received Jan. 16, 1989; revision received Dec. 14, 1989.

*Research Associate, Department of Mechanical and Aerospace Engineering.

†Professor, Department of Mechanical and Aerospace Engineering.

Introduction

A PROBLEM of great practical importance that has seen considerable research activity in the recent years is that of the thermal interaction between the wakes arising from isolated heat sources. The practical applications of the thermal interaction between multiple heat sources include cooling of electronic components in electronic systems, positioning of heating elements in furnaces, arrangement of heating or cooling tubes in refrigeration and air conditioning systems, and enclosure fires, as reviewed by Steinberg,[1] Kraus and Bar-Cohen,[2] Quintiere et al.,[3] and others.

In many applications, such as in electronic systems and furnaces, the electronic components or heating elements are located on horizontally or vertically oriented panels. The heat-dissipating components, such as resistors, semiconductor chips, or heating elements can then be treated as heat sources mounted on a surface. A heated body, situated in a stationary medium, gives rise to a natural convection plume or wake that is essentially a buoyant mass of fluid rising above the heated body. As the plume rises upward, it entrains the ambient fluid and its temperature decays, with an increase in the mass flow rate. In the absence of an adjacent surface, free boundary flows occur in the form of a thermal plume. The characteristics of such free boundary flows have been studied by many investigators (see, for instance, papers by Fujii,[4] Gebhart et al.,[5] and Jaluria and Gebhart[6]). However, in practice, heat sources are often located on flat horizontal or vertical surfaces, giving rise to wall plumes. The behavior of wall plumes generated by isolated line, point, and finite-width sources has been investigated by many researchers (see, for instance, the work of

Zimin and Lyakhov,[7] Carey and Mollendorf,[8] and Sparrow et al.[9]). The temperature field above a concentrated heat source on a vertical adiabatic surface has been studied experimentally by Carey and Mollendorf.[8] The surface temperature excess, $T_s - T_\infty$, was found to vary as $x^{-0.77}$, where x is in the direction of the main flow. Therefore, the flow cools more rapidly than a two-dimensional plume but less rapidly than an axisymmetric plume (see Jaluria[10]).

If the ambient fluid is not stationary but has certain flow velocity U_∞, the plume from the heated body is influenced by the ambient flow, and the resulting convective flow is quite complex. In such a situation the heat transfer from the body could be either predominantly forced or free convection. Otherwise, the two modes of convective heat transfer have comparable effects, and the thermal transport process is usually termed as mixed convection.[10,11]

In all of these situations, if a heated body is placed in the wake of another body that is located upstream, it is subjected to ambient temperature and velocity that are different from those encountered by the upstream body. Thus, its heat transfer characteristics are quite different from those that apply in the absence of such a wake. The interaction of natural convection wakes arising from multiple heat sources has been studied experimentally and theoretically by various investigators, such as Lieberman and Gebhart,[12] Pera and Gebhart,[13] Jaluria,[14,15] and Milanez and Bergles.[16] The results indicated that the wake from an upstream source strongly affects the heat transfer from a downstream source, and, depending on the separation distance between the two heat sources, the heat transfer coefficient of the downstream source may be increased or decreased. The problem of mixed convection from multiple heat sources, mounted on a flat vertical surface, has been studied numerically by Jaluria.[17] The temperature of a heat source located downstream in the wake of an upstream source was found to be considerably higher than that when the upstream source was not present.

Not much work has been done on the conjugate heat transfer problem that arises due to the conductive heat loss from the heat sources to the surface on which the sources are located. However, this is an extremely important consideration in practical problems, as discussed by Zinnes[18] and Kishinami and Seki.[19] Also, there has been little work done on the effect of a small, externally induced, or forced, flow velocity that is usually present in many practical systems. For example, many microcomputers have a small fan that creates a weak external flow over the heat-dissipating electronic components.

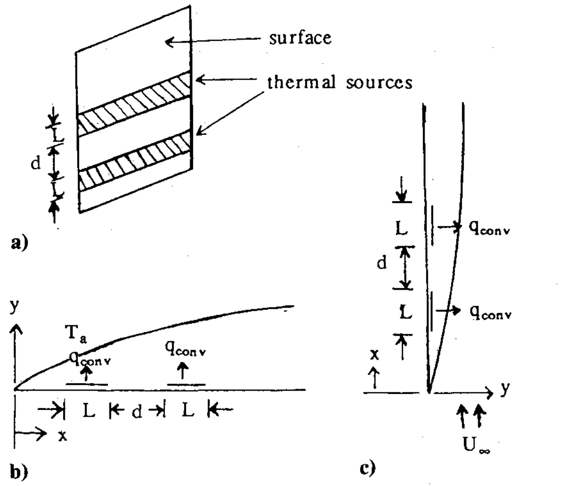

Fig. 1 Schematic of the problem studied: a) schematic drawing; b) coordinate system for the problem studied (horizontal configuration); c) coordinate system for the problem studied (vertical configuration).

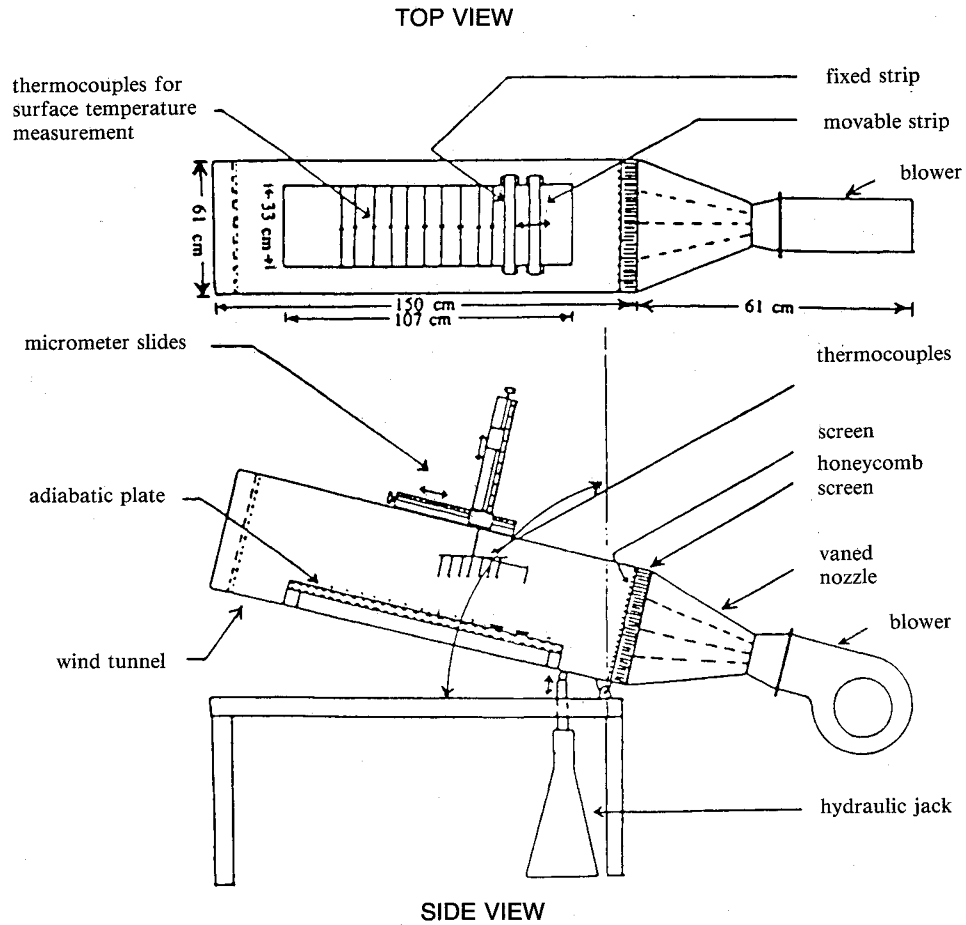

Fig. 2 Top and side views of the experimental system.

The work described in this paper is an experimental study on the fundamental aspects of the thermal interaction of two heat sources mounted on a flat surface, well insulated at the back. The surface and the heat sources are wide in the transverse direction; hence, a two-dimensional flow situation is obtained. The circumstance of natural convection has been studied with and without the presence of a small, externally induced velocity aiding the natural convection. An investigation has been carried out on the effects of the orientation, horizontal and vertical, of the surface and on the conjugate nature of the heat transfer process. Measurements of the flow velocities are made using a specially calibrated hot-wire anemometer. The effect of the upstream heat source on the downstream source is studied by varying the upstream source heating rate and the source separation distance. The effect of a forced flow velocity on the temperatures of isolated heat sources is also investigated. The experimental results are presented in terms of the nondimensional surface temperature θ_s, nondimensional local temperature θ in the flowfield, nondimensional distance from the leading edge of the plate X, and the nondimensional source separation D. This nondimensionalization has been frequently applied to convective flows.[17] The nondimensionalized variables are defined in the Nomenclature and also in the next section. A schematic diagram of the problem studied is shown in Fig. 1.

Experimental Arrangement

The heart of the experimental arrangement is a low-speed wind tunnel with a test section of 61 cm × 46 cm cross-sectional area and 150 cm length. The top and side views of the experimental setup are shown in Fig. 2. The wind tunnel can be inclined at any angle from 0 to 90 deg, with the horizontal, by means of a hydraulic jack. A blower, attached to the tunnel by means of a vaned diffuser, provides a variable flow velocity, ranging from 0 to about 50 cm/s in the test section. The flow from the diffuser enters the test section through a section of honeycomb and a fine-mesh screen. This arrangement provides a fairly uniform flow with a measured turbulence intensity of less than 0.5%. The isolated heat sources are obtained by electrically heating two highly polished stainless steel strips that were 2.54 cm wide and 0.03 mm thick for the results presented here. The heat sources are mounted across a test plate (also referred to as plate or surface in this paper), 107 cm long and 33 cm wide. The plate is placed parallel to the forced flow direction inside the test section of the wind tunnel, as shown. The thermal emissivity of the strips was experimentally found to be of the order of 0.1, and the radiation heat loss to the ambient was estimated to be less than 10% of the total electrical power dissipation.

In order to reduce the conduction losses from the heat sources to the plate, the plate is made of three thin masonite boards, separated by 6-mm air gaps. With this arrangement the Rayleigh number based on the gap height is limited to about 1000; therefore, the heat transfer is primarily due to the conduction through the air gaps. This limits the loss of energy to the plate to about 10% of the total electric heat input.[20] Thus, 80–90% of the input energy goes into the flow. The strips are held taut and in good contact with the plate by specially designed clamps.

Side plates were placed along the edges of the surface in order to prevent a flow in the transverse direction and thus obtain a two-dimensional convective flow circumstance. The leading edge of the plate was made sharp, although different leading-edge geometries were found to have essentially no influence on the variables measured for the small velocity levels considered in this study.

Three high-temperature heat-flux gauges (RdF Corp., model 270310-20) are mounted on the backside of each heating strip to measure the conductive heat flux from the strips to the plate. This heat flux is then subtracted from the uniform heat-flux input due to the electric power dissipation in the strips to obtain the local convective flux, q_{conv}, from the strips. The electric power dissipation in the strips is measured by measuring the voltage across the strips and the current by means of a precision digital voltmeter and ammeter, respectively. The local convective heat fluxes from the plate surface (at locations other than directly under the heating strips and where the convective heat flux is relatively small in magnitude) are measured by high-sensitivity heat-flux gauges (RdF Corp., model 27036-2). It was confirmed that the local heat transfer process was not significantly affected by the presence of these sensors. The experimental error in these measurements is estimated to be around 5% of the measured values.

The surface temperature is measured by a set of 24 individually calibrated, thin thermocouples (copper-constantan, 0.025 mm diam), attached to the surface by a high thermal conductivity cement (Omegatherm 201, Omega, Inc.). The thinness of the thermocouples is essential for minimizing the disturbance to the boundary layer. The temperatures in the flow are measured by a set of seven individually calibrated thermocouples (copper-constantan, 0.050 mm diam), mounted on a rake. The thermocouples are staggered in the direction of the flow to avoid interference between each other. The entire thermocouple rake is mounted on a pair of precision micrometer slides (Velmax, Inc.), which provide the desired horizontal and vertical positioning of the rake. The error in the temperature measurements was estimated to be around 0.1°C, which yielded an inaccuracy of order ±1% in the measured temperature differences.

The flow velocity is measured by a constant-temperature hot-wire anemometer (DISA, model 55D01) with a platinum-plated tungsten wire probe (DISA, model 55P11). Hot-wire anemometry is selected for the flow measurements because of the low velocity levels that exist in the flows of interest in this work. The velocity measuring devices based on pressure drop, such as a pitot tube, cannot be used very accurately because of the extremely low pressure drops induced by these weak flows. The use of a laser Doppler velocimeter is complicated by the seeding requirements in air for these velocity levels.

The low velocity levels and the fact that a thermal field coexists wtih the flowfield require that the hot-wire anemometer to be used in the measurements be calibrated to account for the variable fluid temperature and for the orientation of the flow with respect to the hot-wire sensor. The temperature variations in the flowfield cause changes in the anemometer output that must be distinguished from those due to the velocity. The flow orientation is another important consideration because the flow velocities under study are comparable in magnitude to those in the thermal plume generated by the hot-wire sensor.[10] Under these conditions the orientation of the flow to be measured with respect to the buoyant flow arising from the sensor becomes an important consideration. For example, an externally induced flow in the direction of the sensor plume would cause a buoyancy-aided mixed-convective heat transfer from the sensor wire. However, the circumstance would be that of the buoyancy-opposed mixed-convection transport if the external flow is in a direction opposite to that of the sensor plume. Another circumstance of interest is that of the external flow at an angle with the direction of the plume generated by the hot wire. In all of these circumstances the hot-wire anemometer employed must be calibrated for the orientation used in the actual velocity measurements.

In the present work the effect of a variable fluid temperature was taken into account by a method developed by Hollasch and Gebhart.[21] The method consists of varying the wire overheat during the calibration process at a constant fluid temperature and using an analytically derived expression to relate the anemometer output with variable overheat to the output with variable fluid temperature. A special calibration apparatus was developed to calibrate hot-wire anemometers for variable fluid temperature and for different flow orientations. The details of the setup and the resulting calibration curves are given by Tewari and Jaluria.[22] The error in the velocity measurements reported here is estimated to be less than about 0.5% of the measured velocity.

Experimental Results and Discussion

A detailed experimental study has been carried out on the interaction of the wakes arising from two thermal sources located on a flat surface. The circumstances of the pure natural convection flow and the buoyancy-dominated flow in the presence of a weak forced flow are studied. In a theoretical study on mixed convection from multiple heat sources heated by a uniform heat flux input and mounted on a vertical adiabatic surface, Jaluria[17] obtained $Gr/Re^{5/2}$ as the appropriate mixed-convection parameter for demarcating the regimes of natural, mixed, and forced convective flows. For $Gr/Re^{5/2}$ values significantly less than unity, of the order of unity, and significantly greater than unity, the respective circumstances of forced-flow-dominated, mixed-convection, and buoyancy-dominated flows are obtained. In this paper, if the externally induced velocity is zero, the circumstance is referred to as pure natural convection. In the presence of a relatively small externally induced velocity, so that $Gr/Re^{5/2}$ is considerably greater than unity, the circumstance is termed buoyancy-dominated. Results are presented largely for a typical buoyancy-dominated case ($Gr/Re^{5/2} = 47.0$) and for pure natural convection. The results at other values of $Gr/Re^{5/2}$, particularly for large values of this parameter, were found to be quite similar and are not shown here for conciseness.

In the present work both the vertical and the horizontal orientations of the surface are investigated and the results compared. However, a general comment that applies to both orientations of the surface can be made regarding the surface temperature distribution in the longitudinal direction, i.e., along the surface. Sharp temperature gradients occur near the leading and trailing edges of the heated strips, indicating a substantial diffusion of heat in the longitudinal direction. The significance of this observation is that the problem under consideration is an elliptic one, and boundary-layer assumptions cannot be made in the regions close to the sources.[23] Also, the heating of the plate upstream of the source indicates that the conductive transport in the plate must be considered in conjunction with the convective flow. However, the effect of the longitudinal conduction is limited to about one strip width and is therefore fairly localized. A very close approximation to an adiabatic surface is obtained at distances larger than one strip width downstream of the source.

Convection from Sources on a Vertical Surface

The nondimensionalization used here has been frequently employed in the boundary-layer flows over surfaces heated by a uniform heat flux (see, for instance, the paper by Jaluria[17]). The nondimensionalized distances are defined as

$$X = x/L, \quad X_0 = x_0/L, \quad Y = (y/L)\, Gr^{1/5}, \quad D = d/L \tag{1}$$

The nondimensionalized local and surface temperatures θ and θ_2, respectively, for the pure natural convection circumstance are defined as

$$\theta = (T - T_\infty)\, Gr^{1/5}/(q_{\text{in}}\, L/k) \tag{2}$$

$$\theta_s = (T_s - T_\infty)\, Gr^{1/5}/(q_{\text{in}}\, L/k) \tag{3}$$

Here, the Grashof number Gr is defined as

$$Gr = g\beta q_{\text{in}}\, L^4/k\nu^2 \tag{4}$$

In the presence of an externally induced velocity U_∞, the nondimensionalized surface temperature θ_s is defined as

$$\theta_s = (T_s - T_\infty)\, Re^{1/2}/(q_{\text{in}} L/k) \tag{5}$$

Similarly, the local dimensionless temperature θ is defined in terms of Re, where

$$Re = U_\infty L/\nu \tag{6}$$

The nondimensional coordinate distance X is defined as given earlier in Eq. (1), but Y is defined as

$$Y = (y/L) Re^{1/2} \tag{7}$$

The preceding nondimensionalization is used to obtain both the natural and forced convection circumstances as U_∞ goes from zero to large values.

The effect of the upstream (lower one for the vertical orientation) strip on the downstream (upper) strip is investigated in two stages. In the first stage, the separation d is varied while their heating rates are kept fixed. Note that the downstream strip is held fixed while the upstream strip is movable. Next, the source heating rate q_{in} is varied while the separation distance between them is kept fixed.

In the study of the effect of the source separation, the nondimensionalized separation distance between the strips D is varied from 0 to 5.0. When the separation between the strips is zero, a single strip with twice the single strip height is ob-

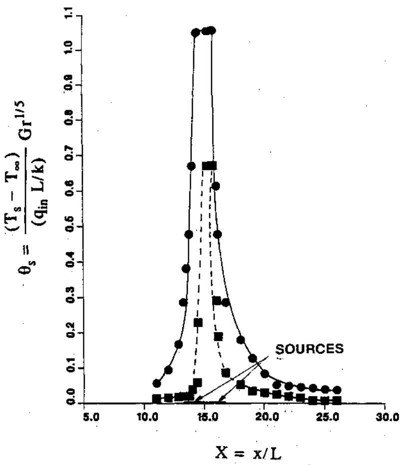

Fig. 3 Surface temperature variation for natural convection from two heat sources on a vertical surface with zero source separation and same heat input; ■, single source (upper); ●, two sources combined to produce a single source. The input heat flux is 1300 W/m².

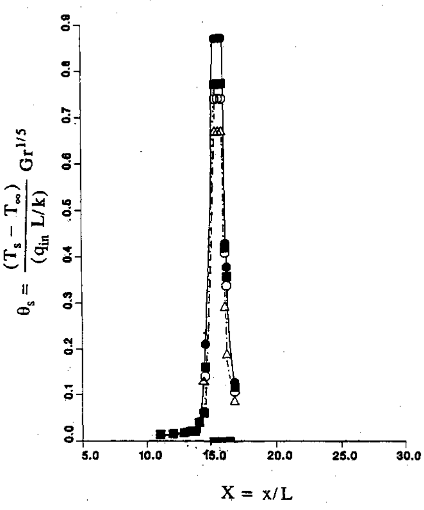

Fig. 4 Effect of the source separation distance on the temperatures of the downstream source on a vertical surface in natural convection flow. Both sources are heated to same heat input; ●, source separation equal to one source width; ■, source separation equal to two source widths; ○, source separation equal to three source widths; △, isolated source. Both sources are at 1300 W/m².

tained. In Fig. 3 a comparison between the measured temperature distribution for a single strip of width L and that for the two combined strips of width $2L$ is shown. The input surface heat flux in both cases is 1300 W/m^2. The temperature levels for the combined strips are seen to be considerably higher than those for the single strip, the maximum θ_s being 1.05 for the combined strips, compared to the corresponding value of 0.7 for the single strip. For the calculations used in this figure the Grashof number for both the circumstances is based on the width for a single source. This is one reason for the higher nondimensional temperature for the combined strips. If the Grashof number for the combined sources is based on twice the single source width, i.e., $2L$, a maximum $\theta_s = 0.9$ is obtained, which is still about 30% higher than the corresponding single strip temperature. This is not surprising since the local heat transfer coefficient decreases with downstream distance due to the increase in the boundary-layer thickness.

As the separation between the strips is increased, the effect of the upstream strip on the downstream strip is expected to decrease. This is evidenced in Fig. 4, where it can be seen that, at a separation of three strip widths, the temperature level of the downstream strip is only about 10% higher than that for an isolated source at the same surface input heat flux. The results shown here are for an input heat flux of 1300 W/m^2. The temperature profiles at the upstream source are not shown for clarity. Similarly, at the source separation distances of L and $2L$, the downstream source temperature is found to be about 30 and 16% higher, respectively, than the corresponding isolated source temperature.

The basic trends observed earlier can be explained as follows. The temperature in the wake decays downstream due to the entrainment of the ambient fluid. In addition, the flow velocity increases downstream due to buoyancy. The onset of turbulence also occurs as the flow proceeds far downstream. Therefore, an increase in d increases the h for the downstream strip, resulting in the observed decrease in the surface temperatures with increasing d. Jaluria[23] found a qualitatively similiar behavior in a theoretical study of the interaction between wakes due to two heated strips on a vertical adiabatic plate in natural convection.

The nondimensional temperatures in the theoretical work[14,23] were found to be two to three times higher than those found in the measurements in the present study. In the theoretical study the entire heat input was assumed to be convected to the flow as a step input over the width of the source. However, in actual practice, a fraction of the electrical heat input is lost due to conduction and radiation. Also, the energy

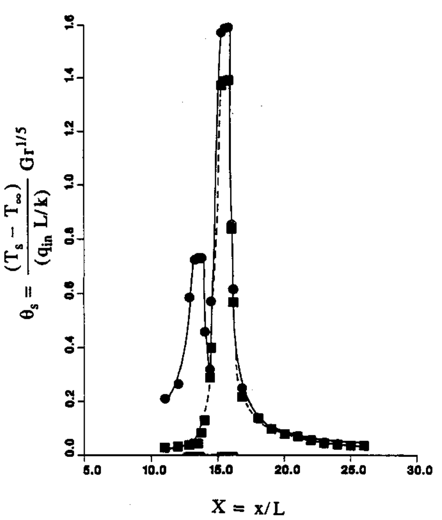

Fig. 5 Effect of the upstream source on the temperature of the downstream source in natural convection; •, upstream source at 1300 W/m^2, downstream source at 3200 W/m^2; ■, upstream source off, downstream source at 3200 W/m^2.

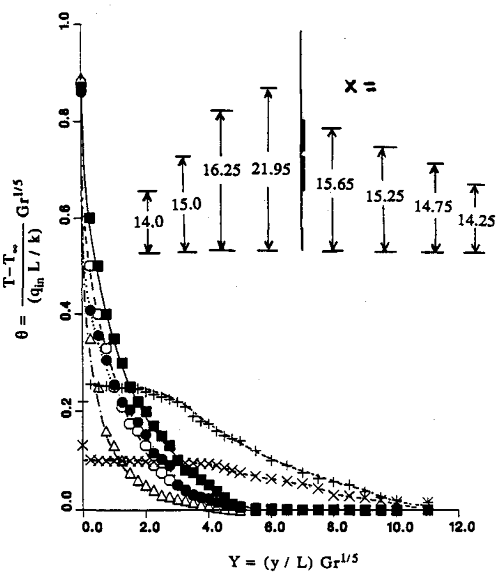

Fig. 6 Temperature profiles in the flow due to natural convection from thermal sources on a flat vertical surface. The source separation distance is zero, and both sources are at 1300 W/m^2; △, $X = 14.25$; ○, $X = 14.75$; •, $X = 15.25$; ■, $X = 15.65$; +, $X = 16.25$; ×, $X = 21.95$.

input occurs over a much wider area due to longitudinal conduction. As mentioned earlier, the conduction loss through the air gaps is only about 10%. However, in the longitudinal direction the conduction is estimated to be about 30% of the heat input because of the large temperature gradients near the leading and the trailing edges of the strips. Thus, despite the thinness of the masonite boards, the conduction along the plate length is significant. However, this energy is eventually transferred to the fluid, though over a much larger area than that of the heat source. In fact, almost 90% of the energy is eventually lost by convection. The longitudinal conduction effects mainly redistribute the energy loss from the source. It is this effect that is dominant in determining the resulting flow and heat transfer, rather than convective and radiative losses that reduce the overall convective transport.

Figure 5 shows how the temperature of the downstream strip is influenced by the upstream strip input heat flux (also referred to as the strength of a heat source in this paper). Here the strip separation is kept fixed at one strip width L. The upstream strip is unheated in one case, and in the second it is heated to an input flux level of 1300 W/m^2. The downstream strip flux is fixed at 3200 W/m^2. The Grashof number used in this figure is based on the heat-flux level of 1300 W/m^2. When the upstream strip is heated at 1300 W/m^2, the temperature of the downstream strip increases by about 15% above the value obtained when the upstream strip was turned off. This situation is referred to as a strong downstream source placed in the wake of a relatively weak upstream source. These results may be compared with the results shown in Fig. 4, where both strips are heated to an equal flux of 1300 W/m^2. In this case the temperature of the downstream source was found to be higher than that without the presence of the upstream strip by about 30% at $D = L$. This is the situation of two sources of equal strength. These results clearly indicate that a weaker upstream source has a smaller effect on the temperature of a relatively stronger downstream source, but a larger effect on a downstream source of about equal strength. As expected, a stronger upstream source has an even larger influence on a relatively weaker downstream source. Indeed, it is found that, when the upstream and downstream strips are heated to flux levels of 3200 and 1300 W/m^2, respectively, the temperature of the downstream strip is about 70% higher than that without the presence of the upstream strip.

In all of the preceding cases, the effect of the presence of a small, externally induced flow velocity U_∞ is to decrease the

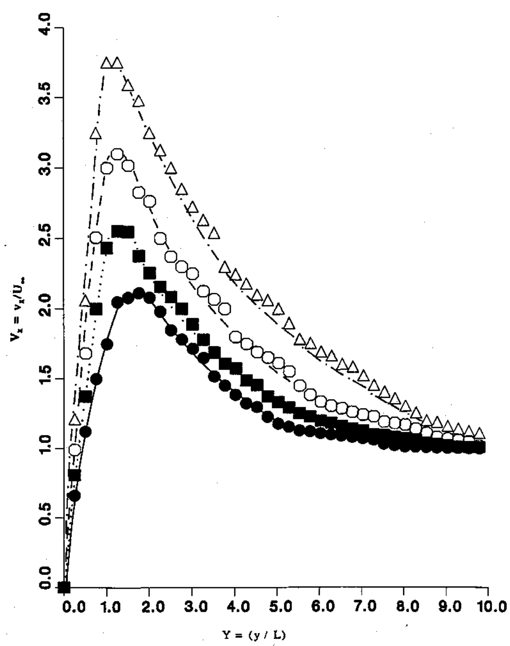

Fig. 7 Flow velocity profiles for the buoyancy-dominated mixed-convection flow ($Gr/Re^{5/2}$ = 47) on a vertical surface. Source separation is one source width; ●, X_0 = 0.5; ■, X_0 = 1.0; ○, X_0 = 2.5; △, X = 3.0.

influence of the upstream source on the downstream one by decreasing the temperature in the wake of the upstream source, as investigated by Tewari and Jaluria.[24] This situation is referred to as a buoyancy-dominated circumstance.

Measurements were also made of the temperature in the flowfield at various y locations away from the surface. The circumstance presented is that of pure natural convection. The nondimensional local temperature θ and distance Y are defined in Eqs. (2) and (1), respectively. The variation of θ with Y from the surface at various X locations is shown in Fig. 6. Both the strips are heated to 1300 W/m^2, and the separation between them is zero. As seen in this figure, sharp temperature gradients occur at the locations adjacent to the heated strips (locations X = 14.25, 14.75, 15.25, and 15.65 in the figure). The temperature gradients decrease rapidly as the flow proceeds downstream (locations X = 16.25 and 21.95 in the figure). Also, the temperature in the fluid adjacent to the heated strips decreases rapidly in the downstream direction because of the entrainment from the ambient medium. Qualitatively similar temperature profiles are obtained at higher and lower heat fluxes, as presented by Goel.[20]

The flow velocities for both the pure natural convection and the buoyancy-dominated circumstances were measured by the specially calibrated hot-wire anemometer, mentioned earlier. For pure natural convection, the nondimensionalized vertical velocity component V_x, which is in the x direction, is defined as[17]

$$V_x = v_x/[(\nu/L)Gr^{2/5}] \tag{8}$$

For the buoyancy-dominated circumstances, which is the case when a small, externally induced velocity U_∞ is present, V_x is defined as

$$V_x = v_x/U_\infty \tag{9}$$

Figure 7 shows the measured velocity profiles for the buoyancy-dominated mixed-convection circumstance at $Gr/Re^{5/2}$ = 47.0. As can be seen, the velocity increases in the downstream direction, the peak value of the velocity moves closer to the surface, and the hydrodynamic boundary layer thickens. The velocity profiles, in general, tend to be smoother for the pure natural convection circumstance (not shown here) than in the case of the buoyancy-dominated flow, where the small, externally induced velocity was found to give rise to flow instability. The physical velocity levels were higher at $Gr/Re^{5/2}$ = 47, compared to the pure natural convection circumstance, because of aiding forced flow and buoyancy effects. The maximum velocity in the measured profile increases in the downstream direction according to the following correlation, derived from the data obtained:

$$V_{x,\max} = 34.94X_0^{0.30} \tag{10}$$

for the case of pure natural convection. For the buoyancy-dominated mixed-convection circumstance, the maximum vertical velocity variation was well correlated by the expression

$$V_{x,\max} = 2.521X_0^{0.26} \tag{11}$$

where X_0 is measured from the lower edge of the upstream strip, since the convective effects start at this location. Note that the definitions of V_x in Eqs. (10) and (11) are different, as given earlier.

The dependence of $V_{x,\max}$ on X may be compared with the $X^{0.20}$ dependence of $V_{x,\max}$ on X in the case of a uniformly heated line source mounted on an adiabatic surface.[10] A comparison may also be made with the velocities calculated by Jaluria[14] for natural convection from two heated strips mounted on a vertical adiabatic surface. The theoretically calculated velocities are found to be about twice the experimentally measured velocities. Again, as in the case of the

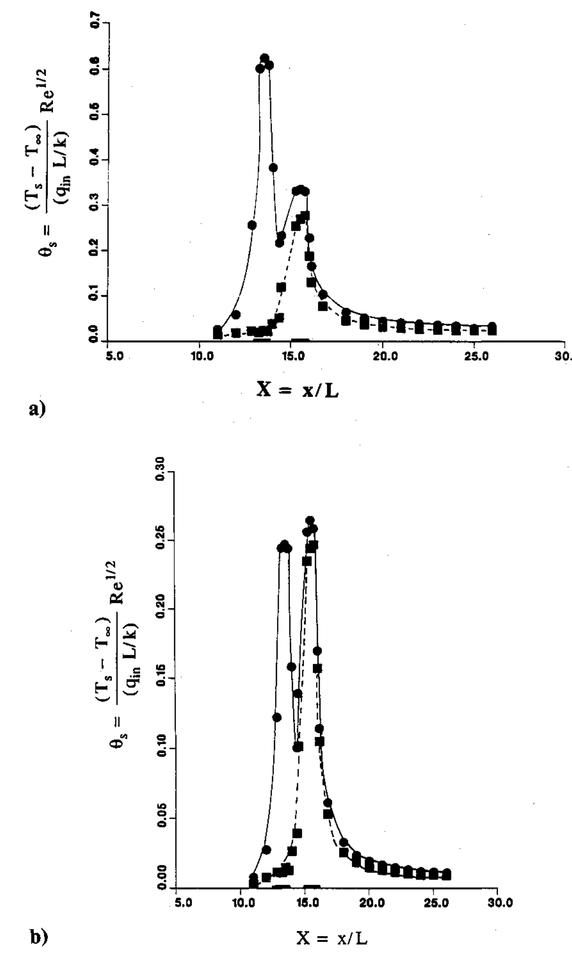

Fig. 8 Surface temperature variation for two thermal sources on a horizontal surface in mixed convection: a) ●, upstream source at 3200 W/m^2; ■, upstream source turned off, downstream source at 1300 W/m^2, $Gr/Re^{5/2}$ = 47, U_∞ = 5 cm/s; b) ●, both sources at 3200 W/m^2; ■, upstream source turned off, downstream source at 3200 W/m^2, $Gr/Re^{5/2}$ = 115, U_∞ = 5 cm/s.

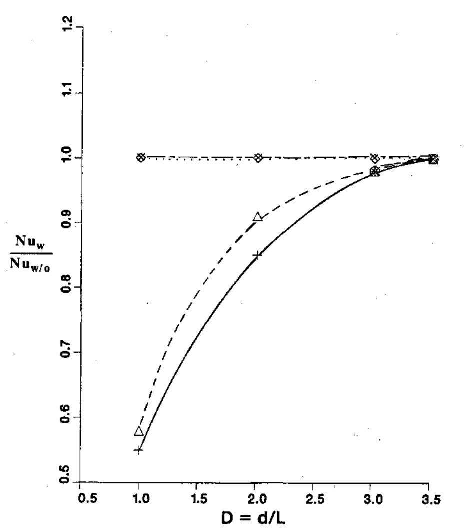

Fig. 9 Effect of a stronger upstream source on the average Nusselt number of a weaker downstream source as a function of the source separation distance; ◇, pure natural convection (horizontal surface orientation); ×, buoyancy dominated ($Gr/Re^{5/2}$ = 47, horizontal surface orientation); +, pure natural convection (vertical surface orientation); △, buoyancy dominated ($Gr/Re^{5/2}$ = 47, vertical surface orientation).

temperature measurements, the low experimental values of velocities can be attributed to the spreading out of the heat input from the sources in the longitudinal direction instead of the step input assumed in the theoretical analysis.

Convection from Sources Located on a Horizontal Surface

The nondimensionalized temperatures for this case are defined in Eqs. (2) and (3). The effect of the upstream strip on the temperature levels at the downstream strip is shown in Fig. 8. The mixed convection parameter, $Gr/Re^{5/2}$, is 47 and 115, and clearly the circumstances lie in the buoyancy-dominated flow regime. The separation between the two strips is kept fixed at one strip width. In Fig. 8a the situation of a weak downstream source located in the wake of a relatively strong upstream source is created by heating the downstream and upstream sources to 1300 and 3200 W/m^2, respectively. The heat flux used here in the calculation of the Grashof number is the smaller of the two input heat fluxes. As seen in Fig. 8, the temperature of the downstream strip is higher than that without the presence of the upstream strip by about 20%.

In the other experiment both of the strips are heated to the same heat-flux level (3200 W/m^2), and the situation of a downstream source placed in the wake of an upstream source of the same strength is obtained, as shown in Fig. 8b. The temperature of the downstream strip is found to be only about 15% higher than that when the upstream strip was turned off. The relative increase, due to the presence of the upstream strip, in the downstream strip temperatures is somewhat smaller than that when the downstream strip is a weaker source, as shown in Fig. 8a. However, the increases in both the cases are much smaller than those in the case of the vertical configuration. Another striking difference between the vertical and horizontal circumstances was found in the case of pure natural convection. It was observed that, in the absence of an externally induced flow velocity in the horizontal orientation, each source was virtually unaffected by the presence of the other for a separation of one source width or larger.

An explanation for the preceding observations is that, in the vertical configuration, the downstream strip is entirely under the influence of the wake of the upstream strip. However, in the horizontal configuration, for the buoyancy-dominated flow, the wake from the upstream strip is a vertically rising plume and has only a limited interaction with the plume rising from the downstream strip. The flow temperatures at the locations between the two strips and at the locations immediately downstream of the downstream strip are found to be significantly lower than the flow temperatures directly above the heated strips. These measurements indicate that the wakes from the strips rise vertically before being dissipated; therefore, the locations downstream of the wakes are not affected. This explanation is further supported by the measurements (by means of heat-flux gauges) of the convective heat flux q_{conv}. When both of the strips are heated to a heat-flux level of 3200 W/m^2, the fraction of the input heat flux convected from the downstream strip remains almost unchanged compared to that when the upstream strip is turned off. Similar results are obtained for a weak source placed downstream of a relatively stronger one. These results clearly indicate that the upstream strip has little influence on the convection from the downstream strip. However, in the presence of even a small, externally induced velocity of 5 cm/s, the wake from the upstream source is tilted in the downstream direction, influencing the downstream locations. This is evidenced by the 10–15% increase in the downstream strip temperature due to the presence of a stronger upstream source.

A further insight into the thermal interaction between the two heated strips can be obtained by measuring the average

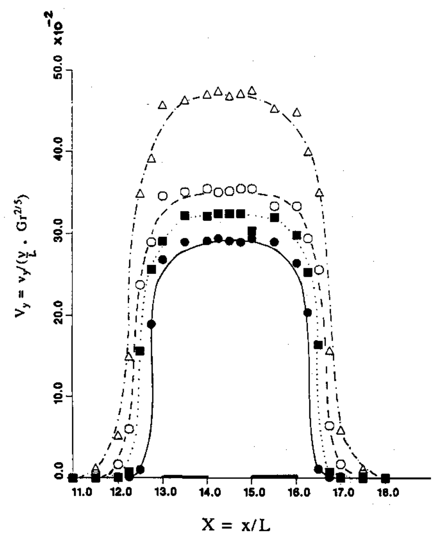

Fig. 10 Vertical velocity profile for natural convection flow arising from two sources on a horizontal surface. Measurements made in the middle plane near an edge of the plate. Source separation is one source width; ●, Y = 0.39; ■, Y = 0.78; ○, Y = 1.56; △, Y = 3.12.

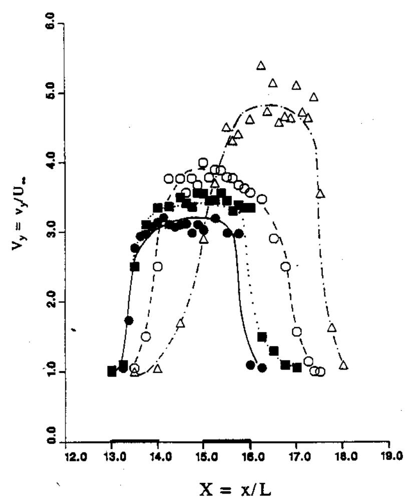

Fig. 11 Vertical velocity profiles for the buoyancy-dominated mixed-convection flow ($Gr/Re^{5/2}$ = 47) on a horizontal surface. Source separation is one source width; ●, Y = 0.30; ■, Y = 0.78; ○, Y = 1.56; △, Y = 3.12.

heat transfer coefficient h for the convection from a heated strip with and without the presence of another heated strip. The heat transfer coefficient is expressed in terms of the Nusselt number as

$$Nu_w \text{ or } Nu_{w/o} = hL/k \tag{12}$$

where h is defined in terms of the average strip surface temperature $\bar{T}_s$ as

$$h = q_{\text{conv}}/(\bar{T}_s - T_\infty) \tag{13}$$

As mentioned earlier, $\bar{T}_s$ and T_∞ are measured by the surface-mounted and rake-mounted thermocouples, respectively. Heat-flux gauges are used to measure q_{conv}.

Figure 9 shows that, for the horizontal orientation of the surface, both pure natural convection and buoyancy-dominated circumstances result in a negligible reduction in the heat transfer coefficient of a heat source due to the presence of another heat source, even when they are separated by only one source width. However, in the vertical orientation a substantial reduction in the heat transfer coefficient of the downstream strip results due to the presence of the heated upstream strip. The influence of the upstream strip decreases as the source separation distance d is increased.

Measurements were also made of the velocities in the flowfield, and the results are presented in terms of nondimensionalized variables. The velocity and distances are nondimensionalized in the same manner as in the vertical orientation so that the two cases may be compared directly. Figure 10 shows the profiles of the vertical velocity, V_y, at various vertical locations over the plate for the pure natural convection circumstance. The nondimensionalized V_y is defined as

$$V_y = v_y/[(\nu/L)\,Gr^{2/5}] \tag{14}$$

The nondimensionalized vertical and horizontal distances from the plate surface are defined in Eq. (1).

It may be noted that the main flow for a thermal plume is in the direction normal to the surface of the plate, i.e., in the y direction. In Figure 10 the measurements shown are made at the middle plane of the plate. As can be seen in the figure, the plumes merge into each other just above the surface. This is evidenced from a relatively uniform velocity, with no distinct peak at $Y = 0.39$, obtained over the region containing the two heated strips. A relatively uniform velocity, instead of a distinct peak over each strip, indicates that the two plumes have merged. The velocity levels obtained are close to those in the plumes from the corresponding isolated heat sources. This indicates that, at this separation, although the plumes merge into each other, the velocity levels are not significantly affected by this interaction.

The maximum value of the velocity $V_{y,\max}$ increases with the vertical distance from the surface according to the following correlation, derived from the results obtained:

$$V_{y,\max} = 34.80\,Y^{0.22} \tag{15}$$

Brodowicz and Kierkus[25] found that the maximum velocity in the natural convection flow from a thin, long, uniformly heated wire increases as $Y^{0.19}$. A comparison may also be made with the maximum velocity in the wall plume, generated by a line source mounted on a vertical adiabatic surface, which has a $Y^{0.20}$ dependence on Y.[10]

Figure 11 shows the plume velocity in the presence of a small, externally induced velocity of 5 cm/s. The nondimensionalized main flow velocity component V_y is defined as

$$V_y = v_y/U_\infty \tag{16}$$

The mixed-convection parameter $Gr/Re^{5/2}$ is 47, and the measurements shown are made at the middle plane of the plate. The plume is seen to be inclined from the horizontal at an angle that changes with the elevation. This is clearly seen from the shifting of the velocity profiles in the direction of the externally induced flow. As discussed earlier, the velocity measurements are made using a hot-wire anemometer calibrated for various orientations of the externally induced flow with the direction of the plume generated by the sensor wire. The orientation of the probe with respect to the main flow component is decided based on the visual observation of the direction of smoke over the heated strips, in a separate experiment. Another important observation is that the locations above the downstream strip are now beginning to be affected by the wake from the upstream source, although the alignment of the upstream wake is not complete yet. In the vertical orientation the upstream wake is always well aligned with the surface, resulting in a much stronger upstream source influence on the downstream locations.

Conclusions

An experimental study of the thermal interaction of the wakes arising from two heat sources mounted on a flat surface has been carried out. The horizontal and vertical surface orientations were studied for both the pure natural convection and the buoyancy-dominated mixed-convection circumstances.

In the vertical orientation of the surface an upstream source is found to have a strong effect on the temperature levels of an equally strong or weaker downstream source, located at a distance up to the order of one source width. At this separation, the downstream source is under the wake of the upstream source and is subjected to a higher ambient temperature, which results in a lower convective heat transfer coefficient and therefore a higher surface temperature. As the wake proceeds farther downstream, its temperature decreases and the velocity increases; therefore, the upstream source has a smaller effect on the downstream source as the separation between them is increased. The effect of the upstream source almost disappears when the source separation distance is increased to three strip widths.

The orientation of the surface is found to have a very strong effect on the interaction between the wakes generated by the two heat sources, in the natural convection circumstance. For instance, in the horizontal orientation of the surface, when the circumstance is that of pure natural convection, which is the case with zero externally induced velocity, a heat source is found to have a negligible effect on the temperature level of another heat source, even when the former is much stronger than the latter. However, in the presence of a small, externally induced velocity, the upstream source begins to influence the temperature level of an equally strong or weaker downstream source. The influence of the upstream source, however, is small compared to that in the corresponding case of the vertical surface orientation. For the case of pure natural convection the observation is attributed to two vertically rising plumes generated by the two heat sources. Though these plumes merge just above the surface, they are not significantly affected by each other. For the case of the buoyancy-dominated flow, the effect on the downstream source is larger, and this observation is attributed to the downstream bending of the upstream wake by the externally induced flow so that the downstream source is brought under the influence of the wake. This situation is similar to that in the vertical configuration, although in the vertical orientation the downstream source is always in the wake of the upstream source. These observations are supported by the flow velocity and the surface heat-flux measurements.

Acknowledgment

The authors acknowledge the support of the National Science Foundation through Grants CBT-84-15364 and CBT-88-03049 for this work.

References

[1]Steinberg, D. S., *Cooling Techniques for Electronic Equipment,* Wiley, New York, 1980.

[2]Kraus, A. D., and Bar-Cohen, A., *Thermal Analysis and Control of Electronic Equipment,* Hemisphere, Washington, DC, 1983.

[3]Quintiere, J. G., Rinkinen, W. J., and Jones, W. W., "The Effect of Room Openings on Fire Plume Entrainment," *Combustion Science and Technology,* 1981, Vol. 26, No. 1, pp. 193–201.

[4]Fujii, T., "Theory of the Steady Laminar Natural Convection Plumes Above a Horizontal Line Source and a Point Source," *International Journal of Heat and Mass Transfer,* Vol. 6, No. 5, 1963, pp. 597–606.

[5]Gebhart, B., Pera, L., and Schorr, A. W., "Steady Laminar Natural Convection Plumes Above a Horizontal Line Heat Source," *International Journal of Heat and Mass Transfer,* Vol. 13, No. 1, 1970, pp. 161–171.

[6]Jaluria, Y., and Gebhart, B., "On the Buoyancy-Induced Flow Arising from a Heated Hemisphere," *International Journal of Heat and Mass Transfer,* Vol. 18, No. 3, 1975, pp. 415–431.

[7]Zimin, V. D., and Lyakhov, Y. N., "Convective Wall Plume," *Journal of Applied Mechanics and Technical Physics (USSR),* Vol. 11, No. 1, 1970, pp. 159–162.

[8]Carey, V. P., and Mollendorf, "The Temperature Field Above a Concentrated Heat Source on a Vertical Adiabatic Surface," *International Journal of Heat and Mass Transfer,* Vol. 20, No. 20, 1977, pp. 1059–1067.

[9]Sparrow, E. M., Patankar, S. V., and Abdel-Wahed, R. M., "Development of Wall and Free Plumes Above a Heated Vertical Plate," *Journal of Heat Transfer,* Vol. 100, No. 1, 1978, pp. 184–190.

[10]Jaluria, Y., *Natural Convection Heat and Mass Transfer,* Pergamon, Oxford, UK, 1980.

[11]Gebhart, B., *Heat Transfer,* 2nd ed., McGraw-Hill, New York, 1971.

[12]Leiberman, A., and Gebhart, B., "Interactions in Natural Convection from an Array of Heated Elements," *International Journal of Heat and Mass Transfer,* Vol. 12, No. 11, 1969, pp. 1385–1396.

[13]Pera, L., and Gebhart, B., "Laminar Plume Interactions," *Journal of Fluid Mechanics,* Vol. 68, No. 2, 1975, pp. 259–272.

[14]Jaluria, Y., "Buoyancy-Induced Flow due to Isolated Thermal Sources on a Vertical Surface," *Journal of Heat Transfer,* Vol. 104, No. 2, 1982, pp. 223–227.

[15]Jaluria, Y., "Natural Convection Due to Line Sources on a Vertical Adiabatic Surface," *Proceedings of the Seventh International Heat Transfer Conference,* Vol. 2, Hemisphere, New York, 1982, pp. 147–152.

[16]Milanez, L. F., and Bergles, A. E., "Studies on Natural Convective Heat Transfer from Thermal Sources on a Vertical Surface," *Proceedings of the Eighth International Heat Transfer Conference,* Vol. 3, Hemisphere, New York, 1986, pp. 1347–1352.

[17]Jaluria, Y., "Mixed Convection Flow over Localized Multiple Thermal Sources on a Vertical Surface," *Physics of Fluids,* Vol. 29, No. 4, 1986, pp. 934–940.

[18]Zinnes, A. E., "The Coupling of Conduction with Laminar Natural Convection from a Vertical Flat Plate with Arbitrary Surface Heating," *Journal of Heat Transfer,* Vol. 92, No. 3, 1970, pp. 528–535.

[19]Kishinami, K., and Seki, N., "Natural Convection Heat Transfer on an Unheated Vertical Plate Attached to an Upstream Isothermal Plate," *Journal of Heat Transfer,* Vol. 105, 1983, pp. 759–766.

[20]Goel, S., "An Experimental Study of Non-Boundary Layer Effects due to Finite Sized Thermal Sources on a Flat Unheated Surface," Masters Thesis, Dept. of Mechanical and Aerospace Engineering, Rutgers—The State University of New Jersey, Dec. 1984.

[21]Hollasch, K., and Gebhart, B., "Calibration of Constant Temperature Hot Wire Anemometers at Low Velocities in Water with Variable Fluid Temperature," *Journal of Heat Transfer,* Vol. 94, No. 1, 1972, pp. 17–22.

[22]Tewari, S. S., and Jaluria, Y., "Calibration of Hot-Wire Anemometers for Low Velocities in Air," *Review of Scientific Instruments,* Vol. 61, No. 12, 1990, pp. 3834–3845.

[23]Jaluria, Y., "Interaction of Natural Convection Wakes Arising from Thermal Sources on a Vertical Surface," *Journal of Heat Transfer,* Vol. 107, No. 4, 1985, pp. 883–892.

[24]Tewari, S. S., and Jaluria, Y., "Mixed Convection from Thermal Sources Mounted on Horizontal and Vertical Surfaces," *Journal of Heat Transfer,* Vol. 112, No. 4, 1990, pp. 975–987.

[25]Brodowicz, K., and Kierkus, W. T., "Experimental Investigation of Laminar Free-Convection in Air Above Horizontal Wire with Constant Heat Flux," *International Journal of Heat and Mass Transfer,* Vol. 9, 1966, pp. 81–94.